# Measures of Variation

Getting the Idea

Instead of describing the center of a set of data by using the measures of central tendency, you may wish to describe the spread of a set of data. Measures of variation show how spread out or close together the data in a set are, or how much the data points vary.

When data are arranged from least to greatest, the median divides the data into two equal halves. The **first quartile** is the median of the data values that are less than the median. The **third quartile** is the median of the data values that are greater than the median. The **quartiles** and the median divide the data into four quarters. The **range** is the difference between the greatest value in a data set and the least value. The **interquartile range (IQR)**, is the difference between the third quartile and the first quartile. The range measures the spread of all the data. The IQR measures the spread of the two middle quarters of the data.
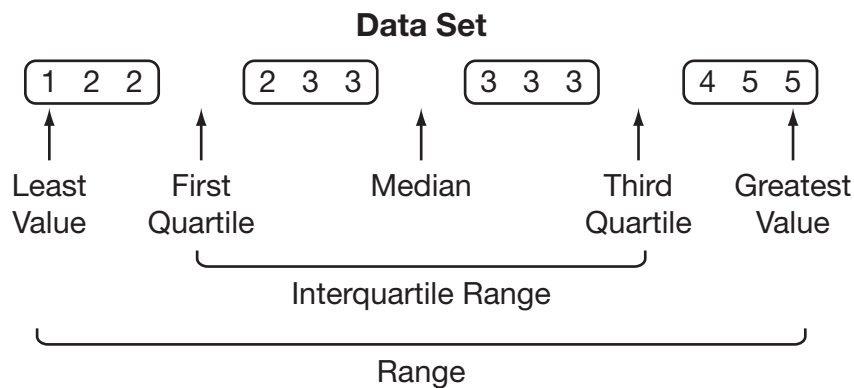
Data Set

For the data set in the diagram, the median is 3. The first quartile is 2 (the mean of 2 and 2). The third quartile is 3.5 (the mean of 3 and 4). The range is $5 - 1$, or 4. The IQR is $3.5 - 2$, or 1.5.

In a data set, a number that is much less or much greater than the other numbers in the data set is an **outlier**. A data set may contain one or more outliers. An outlier will affect the range, but it will not affect the interquartile range. This is why the median is not as affected by outliers as the mean is.

Consider the following data set: 2, 3, 5, <u>6</u>, 8, 10, 23.

The median is 6.

The first quartile is 3, which is the median of the values 2, <u>3</u>, 5.

The third quartile is 10, which is the median of the values 8, <u>10</u>, 23.

The range is 21: $23 - 2 = 21$.

The IQR is 7: $10 - 3 = 7$.

For this data set, 23 is an outlier. It is much greater than the other numbers in the set. The range is affected by this outlier. It suggests that there is a greater variability in the data, since it shows a greater spread, than the IQR suggests. The data vary by 21 from the least to greatest values, while they only vary by 7 away from the median, or the center of the data.

Example 1

The dot plot below shows the grades students received on a grammar test in Ms. Parsi's class.

Grammar Test Grades
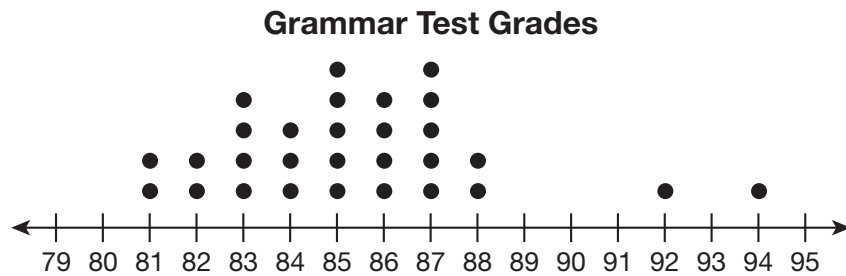
What is the range in the grades? What is the median grade?

Strategy   **Use the dot plot.**

Step 1   Find the greatest and least test grades.

The lowest grade is 81.

The highest grade is 94.

Step 2   Find the range.

Subtract the lowest grade from the highest grade.

range $= 94 - 81 = 13$

Step 3   Count the total number of test grades.

There are 29 grades.

Step 4   Find the median.

The median is the middle grade. It is halfway between the lowest and the highest grade.

Since there are 29 grades, there are 14 grades below the median and 14 grades above the median.

The median is the fifteenth grade in the ordered list of grades.

Start at 81 and count the dots until you reach the fifteenth dot.

This dot is at the grade of 85. So the median is 85.

**Solution**   **The range in the grades is 13, and the median is 85.**

A **box plot**, also known as a box-and-whiskers plot, is a method of visually displaying a distribution of data values by using the median, quartiles, and extremes (least and greatest values) of the data set. The box shows the middle 50% of the data.
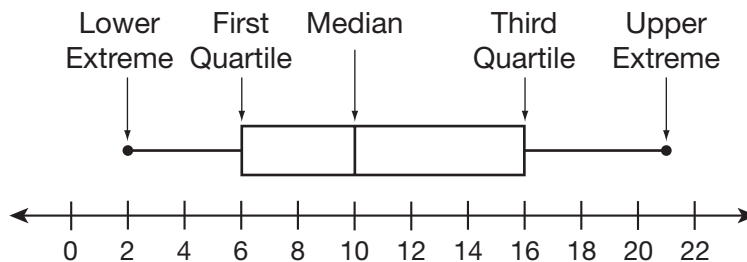
A box plot is a good way to show the spread of a set of data.

Example 2

The box plot below shows the ages of Mr. Morehouse's grandchildren.

Ages of Grandchildren
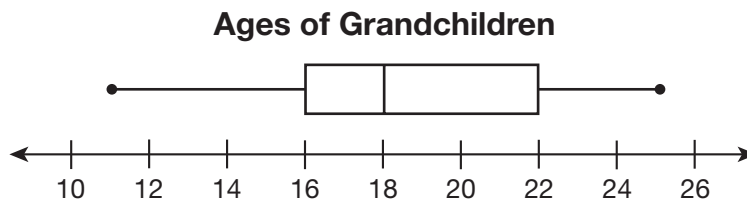
Find the median, first quartile, third quartile, and interquartile range of their ages.

Strategy   Use the box plot.

Step 1   Find the median.

The median is the middle value. On a box plot, it is represented by the vertical line inside the box.

The vertical line inside the box is above 18. The median is 18.

Step 2   Find the first quartile.

On a box plot, the first quartile is represented by the box's left vertical line.

The left vertical line is above 16. The first quartile is 16.

Step 3   Find the third quartile.

On a box plot, the third quartile is represented by the box's right vertical line.

The right vertical line is above 22. The third quartile is 22.

Step 4   Find the interquartile range.

IQR = third quartile − first quartile = 22 − 16 = 6

Solution   **The median of the ages is 18. The first quartile is 16. The third quartile is 22. The interquartile range is 6.**

Example 3

Below are the quiz scores from students in two different class sections.

Section 1: 7, 9, 9, 10, 8, 6, 8, 5, 5, 9, 10, 7, 8, 7, 9

Section 2: 7, 8, 9, 9, 8, 8, 7, 9, 9, 10, 8, 8, 7, 10, 8

Which section has greater variability in the scores?

Strategy	Compare the ranges and interquartile ranges of the two class sections.

Step 1   Find the range for Section 1.

Order the scores from least to greatest.

5, 5, 6, 7, 7, 7, 8, 8, 8, 9, 9, 9, 9, 10, 10

The highest score was 10. The lowest score was 5.

Range = 10 − 5 = 5

Step 2   Find the median and the quartiles for Section 1.

5, 5, 6, <u>7</u>, 7, 7, 8, <u>8</u>, 8, 9, 9, <u>9</u>, 9, 10, 10

The median is 8, the first quartile is 7, and the third quartile is 9.

Step 3   Find the interquartile range for Section 1.

The interquartile range (IQR) is the difference between the third and first quartiles.

IQR = 9 − 7 = 2

Step 4   Find the range for Section 2.

Order the scores from least to greatest.

7, 7, 7, 8, 8, 8, 8, 8, 8, 9, 9, 9, 9, 10, 10

Range = 10 − 7 = 3

Step 5   Find the median and the quartiles for Section 2.

7, 7, 7, <u>8</u>, 8, 8, 8, <u>8</u>, 8, 9, 9, <u>9</u>, 9, 10, 10

The median is 8, the first quartile is 8, and the third quartile is 9.

Step 6   Find the interquartile range for Section 2.

IQR = 9 − 8 = 1

Step 7   Compare the range and interquartile range for each section.

The range for Section 1 is 5, and the IQR is 2.

The range for Section 2 is 3, and the IQR is 1.

The range and IQR for Section 1 are greater than for Section 2.

Solution	**Section 1 has greater variability in the quiz scores than Section 2.**

Example 4

The double box-and-whisker plot below shows the number of points scored in games by two basketball players on the same team.

Points Scored in Games
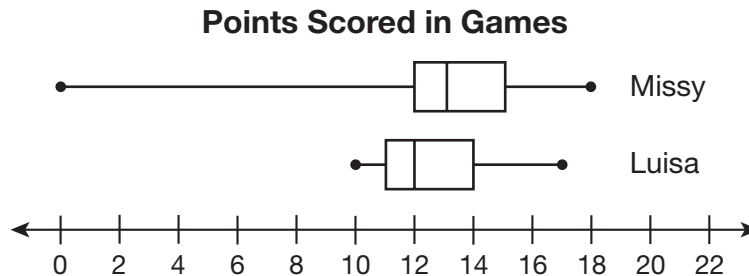
Find the range and the interquartile range for each player. Who is the more consistent scorer?

Strategy   **Compare the range and interquartile range for each player.**

Step 1   Find the range and the interquartile range for Missy.

Missy's highest score was 18. Her lowest score was 0.

Range = 18 − 0 = 18

The interquartile range (IQR) is the difference between the values at the ends of the box, or between the third and first quartiles.

IQR = 15 − 12 = 3

Step 2   Find the range and the interquartile range for Luisa.

Luisa's highest score was 17. Her lowest score was 10.

Range = 17 − 10 = 7

The third quartile score for Luisa is 14, and the first quartile score is 11.

IQR = 14 − 11 = 3

Step 3   Who is the more consistent scorer?

Missy's scores per game show a much greater range than Luisa's.

Notice that Missy's scores have an outlier. She scored 0 points during one game.

The outlier affects the range of her scores.

The interquartile range for both players is the same.

Solution   **Luisa may be a slightly more consistent scorer. Since the interquartile ranges for Luisa and Missy are the same, it is likely that Missy is about as consistent a scorer as Luisa.**

Coached Example

The double box-and-whisker plot below shows the weights, in pounds, of Labrador Retrievers and Cocker Spaniels from a veterinarian's office.

Dog Weights (in pounds)
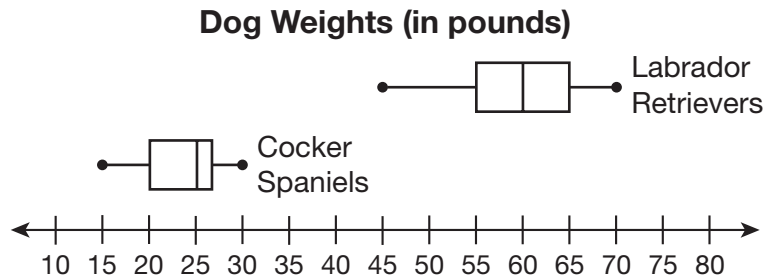
Which type of dog shows greater variability in weight?

The least weight for the Cocker Spaniels is _____ pounds.

The greatest weight for the Cocker Spaniels is _____ pounds.

Find the range of the Cocker Spaniels' weights: _____ − _____ = _____.

The range of the weights of the Cocker Spaniels is _____ pounds.

The third quartile weight for the Cocker Spaniels is _____ pounds.

The first quartile weight for the Cocker Spaniels is _____ pounds.

Find the IQR for the Cocker Spaniels' weights: 27 − _____ = _____.

The IQR for the weights of the Cocker Spaniels is _____ pounds.

The least weight for the Labrador Retrievers is _____ pounds.

The greatest weight for the Labrador Retrievers is _____ pounds.

Find the range of the Labrador Retrievers' weights: _____ − _____ = _____.

The range of the weights of the Labrador Retrievers is _____ pounds.

The third quartile weight for the Labrador Retrievers is _____ pounds.

The first quartile weight for the Labrador Retrievers is _____ pounds.

Find the IQR for the Labrador Retrievers' weights: _____ − _____ = _____.

The IQR for the weights of the Labrador Retrievers is _____ pounds.

The range and the IQR for Cocker Spaniels are _____ than for Labrador Retrievers.

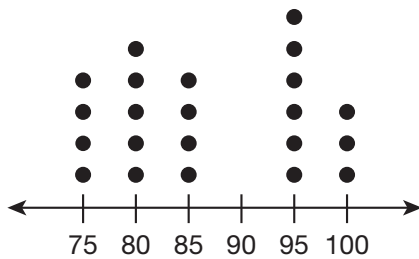_____ **have greater variability in weight**

than _____.

Lesson Practice

Choose the correct answer.

Use the dot plot below for questions 1 and 2.

The dot plot below shows the grades that a class of students received on their recent social studies homework assignment.
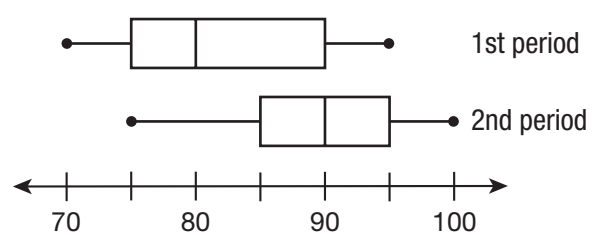
Homework Assignment Grades

75 80 85 90 95 100

1.   What is the first quartile grade?

   A. 75
   B. 80
   C. 85
   D. 90

2.   What is the third quartile grade?

   A.   85
   B.   90
   C.   95
   D. 100

Use the double box-and-whisker plot for questions 3 and 4.

The double box-and-whisker plot below shows the vocabulary quiz scores for Mr. Edelman's first and second period classes.

Quiz Scores

1st period

2nd period

70   80   90   100

3.   What is the interquartile range of the first period quiz scores?

   A.   5
   B.   10
   C.   15
   D.   20

4.   Which statement about the quiz scores is true?

   A.   The range of the scores was the same for both classes.

   B.   The interquartile range of the scores was the same for both classes.

   C.   The mean score was the same for both classes.

   D.   About 25% of the students in both classes scored 95 or higher on the quiz.
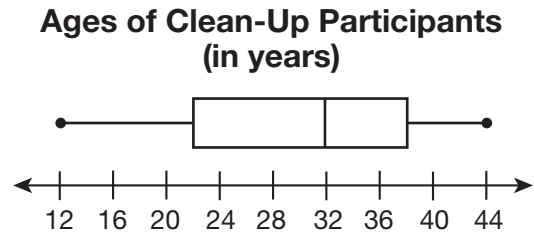
5. The box-and-whisker plot shows the number of miles run per week by the members of a running club.

Miles Run Per Week
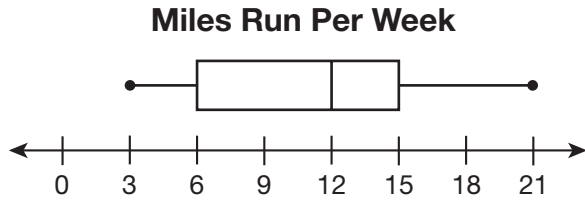
What is the range of the data?

A. 9 miles

B. 12 miles

C. 15 miles

D. 18 miles

6. The box-and whisker plot shows the ages of the participants in a park clean-up.

Ages of Clean-Up Participants (in years)

What is the interquartile range of the ages, in years?

A. 12

B. 16

C. 20

D. 32

7. The double box-and-whisker plot below shows the heights, in millimeters, of plants that Jerand and Marsha grew for a science project.

Plant Heights (in millimeters)
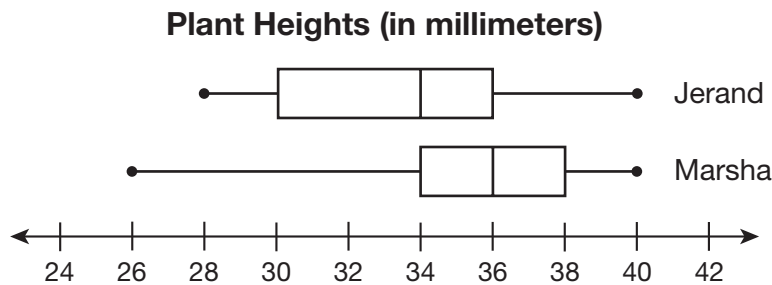
A. How do the range and the interquartile range for Jerand's plants compare to the range and the interquartile range for Marsha's plants? Show your work.

_____

_____

_____

B. Whose plants show greater variability in height? Explain your thinking.

_____

_____

_____

8. Pooja kept track of the number of miles she biked each day for a month. Her results are shown in the box-and-whisker plot shown below. Find the measures of central tendency and the measures of variation. Draw a line from each measure to its value.

Miles Biked
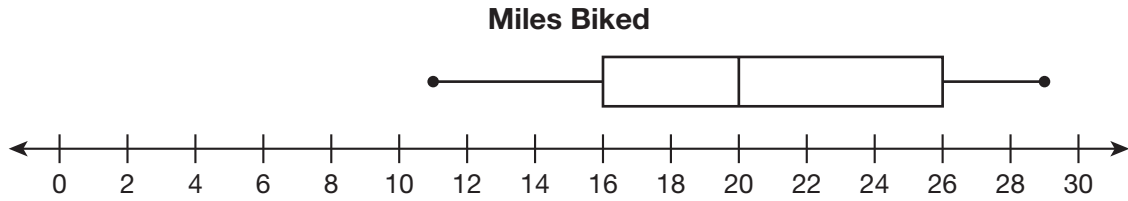
A. least value ●   ● 10

B. first quartile ●   ● 11

C. median ●   ● 16

D. third quartile ●   ● 18

E. greatest value ●   ● 20

F. interquartile range ●   ● 26

G. range ●   ● 29

9. The manager of an animal shelter keeps track of the weight of each dog available for adoption. The dot plot shows the results. Select True or False for each statement.

Dogs' Weight (in pounds)
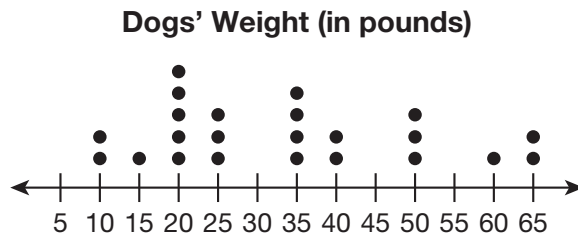
A. The values of the IQR and the first quartile are equivalent.   ○ True   ○ False

B. The range of the dogs' weights is 55 pounds.   ○ True   ○ False

C. The third quartile of the data is 20 pounds.   ○ True   ○ False

D. The median weight of the dogs is 35 pounds.   ○ True   ○ False

10. Elena and Charlie are in the Student Leaders Club. They kept track of the number of service hours they completed each month. The results are shown in the box-and-whisker plots shown below. Which is a true statement? Circle all that apply.

Service Hours Completed
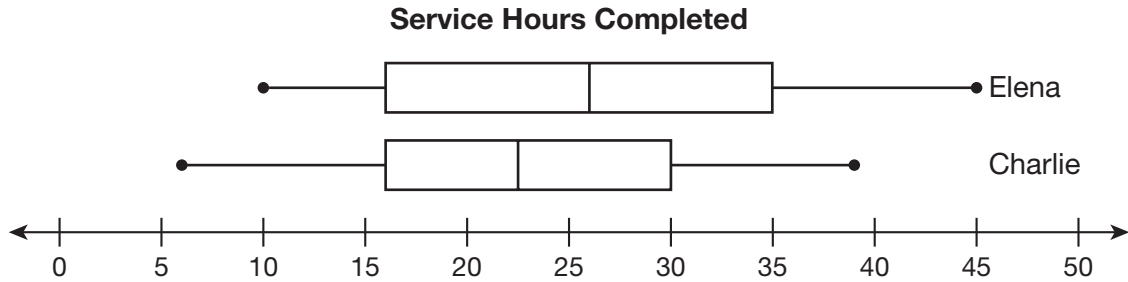
A. Elena's IQR is greater than Charlie's IQR.

B. The medians of both sets of data are equal.

C. Charlie's range is less than Elena's range.

D. Elena's greatest number of service hours is less than Charlie's greatest number of service hours.

11. Compare the two box-and-whisker plots of data. Find the values for the first quartiles, third quartiles, and interquartile ranges. Write each value in the correct box.

Money Saved (in dollars)
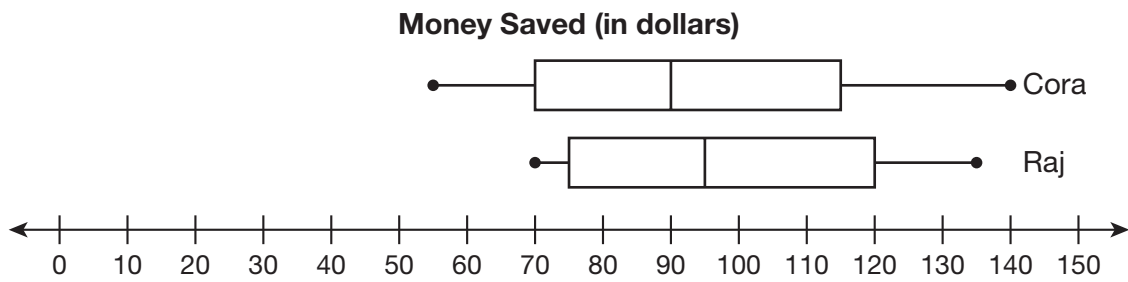
| 70 | 115 | 45 | 75 | 120 |

First Quartile	Third Quartile	Interquartile Range